

**Qualifying Exam
Real Analysis**

May, 2007

Name: _____ ID# _____

ATTENTION! Do any 6 of the following 7 problems. Circle the problems you submit for grading.

1. State and prove Egoroff Theorem.

2. State and prove Riesz-Fischer Theorem.

3.

a). State and prove Kakutani-Krein Theorem.

b). State and prove Stone-Weierstrass theorem.

4.

a). State and prove Hahn Decomposition Theorem.

b). State and prove Jordan Decomposition Theorem.

5. State and prove Riesz Representation Theorem for the dual of $L^p(X)$, $1 \leq p < \infty$.

6. Prove: If X is a compact metric space and $F \in [C(X)]^*$ is a positive linear functional, then there exists a nonnegative Borel measure μ on X such that

$$F(f) = \int_X f d\mu$$

for any $f \in C(X)$.

7. Let X be a compact metric space. State and prove Riesz-Markov Theorem for the dual of $C(X)$.